

现代物理的入门, QFT 必不可少,

教师: 邵鼎煜, S517, dyshao@fudan.edu.cn

40 + 60
HW Finals

参考书:

<< An Introduction to Quantum Field Theory >>

M. Peskin, Schroeder

Part I: Feynman Diagrams and QED

<< Quantum field theory and the standard model

M. Schwartz Part I & II.

<< Quantum field theory >> Srednicki

<< The quantum theory of fields, I >> S. WEINBERG

讨论班, 周二下午 3:30 - 6:00

江湾 S140.

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

—Richard P. Feynman

科学方法论是研究自然规律的基本手段,它通过以下几个步骤不断拓展人类认知的边界。

- 总结现象规律: 观察自然界的现象, 并从现象中总结现象规律;
- 构建公理化理论体系: 提炼假说, 并通过自洽的数理逻辑推导构建理论体系;
- 做出预言: 在已构建的理论体系下, 对新的实验现象进行定量预言;
- 实验检验: 通过实验, 检验在新理论体系下推导得到的预言;
- 证实或证伪: 如果实验检验证实了预言, 则说明假说可以被接受; 如果不符或者发现了新的不能被已有理论解释的现象, 则修正假说, 并重新推导预言进行实验检验。

这是一个循环上升的过程。科学方法论阐述了科学研究的内在逻辑, 使得科学可以被理解和利用, 也促进了技术的进步; 而技术的进步反过来也推动了实验手段的发展, 让人类对自然界的探索不断深入。

在过去的一百多年中, 从电子和原子核, 到组成原子核的质子和中子, 再到组成质子和中子的夸克, 这些微观粒子及它们之间的强、弱和电磁相互作用逐步被发现和研究, 人类对自然界的基本组成和基本相互作用的认识取得了极大的成功。一方面, 正是实验手段的不断进步促成了这些发现; 而另一方面, 人类对于微观系统动力学的认识, 也从相对论、量子力学过渡到了量子场论, 并且在量子场论的框

架下对一些可观测量给出了高精度的预言, 并被更进一步的实验验证。其中, 人们对轻子反常磁矩的理论预言在 10^{-9} 精度上与实验结果吻合^①, 是迄今为止实验验证精度最高的理论预言。

需要特别注意的是, 科学是可以反直觉的。我们对自然界的直观常识 (common sense) 只是来自一定条件下的特定尺度, 即低速宏观条件下的现象, 因此这些常识也只适用于宏观低速的系统, 并不具有普适性。事实上, 高速和微观条件下的物理现象在一定程度上都是反常识的, 本章中光和电子的“波粒二象性”及原子核的衰变都是典型的反直觉现象。而科学家们能建立起相对论和量子力学这样反常识的科学理论, 靠的也并非天马行空的想象, 而是从实验观测到理论推演再到实验检验这样一套完整严谨的科学方法论。

Introduction: Why Quantum Field Theory

The concept of wave-particle duality tells us that the properties of electrons and photons are fundamentally very similar.

Electron : the matter particle ; one elementary constituents of Nature

Photon : a ripple of the electromagnetic field

If they are truly to be placed on equal footing, how should we reconcile this difference in the quantum world ?

- ① View particle as fundamental ; the EM field arising only in some classical limit from a collection of quantum photons
- ② View the field as fundamental ; the photon appears only when we correctly treat the EM field with quantum theory consistently ;

If it is correct , should we introduce an "electron field" ?

The 2nd viewpoint above is the most useful:

The field is primary and particles are derived concepts,
appearing only after quantization.

In this course we will show how photons arise from the quantization of the EM field, how massive , charged particles such as electrons arise from the quantization of matter fields.

Also. quark fields, neutrino fields, gluon fields, W and Z-boson fields, Higgs fields

In classical physics, the primary reason for introducing the concept of the field is to construct laws of Nature that are "local".

The old laws of Coulomb and Newton involve "action at a distance"

⇒ If a distant proton (or star) moves, then the force felt by an electron (or planet) changes immediately.

The above situation is experimentally wrong.

The requirement of locality remains a strong motivation for studying field theories in the quantum world.

Why QFT ?

① Special relativity implies that particle number is not conserved.

Any attempt to write down a relativistic version of the one-particle Schrödinger equation is doomed to failure.

E.g. Negative probabilities, infinite towers of negative energy states, a breakdown in causality.

② All particles of the same type are the same.

Quantum particles that are the same are truly indistinguishable.

Swapping two particles leaves the state unchanged, up to a possible minus sign that determines the statistics of the particle.

In QM, you have to put these statistics in by hand.

In QFT, it is a consequence of the framework.

QFT is the quantization of a classical field. In standard QM, taking the classical d.o.f. and promote them to operators acting on a Hilbert space.

systems with infinitely many degree of freedom, e.g. the classical electro-magnetic field $A^\mu(x)$

Quantum theory of a single particle, e.g. the wave function of an electron. $\psi(x)$

Quantisation

compatibility with relativistic principle

Quantum Field Theory

the basic d.o.f. $\phi(\vec{x}, t)$ field operator

- The possible interactions in QFT are governed by a few basic principles:

- ① locality
- ② symmetry
- ③ renormalization group flow

- According to present knowledge, QFT is the conceptual framework for all microphysical processes.

- QFT is the most important achievement of theoretical physics in the second half of the 20th century.

- Indispensable for modern particle physics, but also for many condensed matter, cosmology, quantum gravity and pure mathematics.

- High energies $E \gg m$
 $\Rightarrow$ particle production
 (e.g. e^+e^- pairs)
 $\Rightarrow$ infinitely many d.o.f.
- Relativistic causality:
 events at space-like separation
 $(x-y)^2 < 0$, cannot influence each other.

$$\frac{g-2}{2} = 0.0011596522 \dots$$

Although the methods developed in this course are general, it focuses on relativistic QFT (i.e. theories obeying Lorentz invariance). The combination of the quantum and relativity principles is particularly powerful in constraining consistent theories.

Notations and Conventions

Units

We will work in “God-given” units, where

$$\hbar = c = 1.$$

In this system,

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{mass}]^{-1}.$$

The mass (m) of a particle is therefore equal to its rest energy (mc^2), and also to its inverse Compton wavelength ($mc/\hbar$). For example,

$$m_{\text{electron}} = 9.109 \times 10^{-28} \text{ g} = 0.511 \text{ MeV} = (3.862 \times 10^{-11} \text{ cm})^{-1}.$$

A selection of other useful numbers and conversion factors is given in the Appendix.

Relativity and Tensors

Our conventions for relativity follow Jackson (1975), Bjorken and Drell (1964, 1965), and nearly all recent field theory texts. We use the metric tensor

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

with Greek indices running over 0, 1, 2, 3 or t , x , y , z . Roman indices— i , j , etc.—denote only the three spatial components. Repeated indices are summed in all cases. Four-vectors, like ordinary numbers, are denoted by light italic type; three-vectors are denoted by boldface type; unit three-vectors are denoted by a light italic label with a hat over it. For example,

$$x^\mu = (x^0, \mathbf{x}), \quad x_\mu = g_{\mu\nu} x^\nu = (x^0, -\mathbf{x});$$

$$p \cdot x = g_{\mu\nu} p^\mu x^\nu = p^0 x^0 - \mathbf{p} \cdot \mathbf{x}.$$

A massive particle has

$$p^2 = p^\mu p_\mu = E^2 - |\mathbf{p}|^2 = m^2.$$

Note that the displacement vector x^μ is “naturally raised”, while the derivative operator,

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \nabla \right),$$

is “naturally lowered”.

We define the totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ so that

$$\epsilon^{0123} = +1.$$

Be careful, since this implies $\epsilon_{0123} = -1$ and $\epsilon^{1230} = -1$. (This convention agrees with Jackson but not with Bjorken and Drell.)

Quantum Mechanics

We will often work with the Schrödinger wavefunctions of single quantum-mechanical particles. We represent the energy and momentum operators acting on such wavefunctions following the usual conventions:

$$E = i \frac{\partial}{\partial x^0}, \quad \mathbf{p} = -i \nabla.$$

These equations can be combined into

$$p^\mu = i \partial^\mu;$$

raising the index on ∂^μ conveniently accounts for the minus sign. The plane wave $e^{-ik \cdot x}$ has momentum k^μ , since

$$i \partial^\mu (e^{-ik \cdot x}) = k^\mu e^{-ik \cdot x}.$$

The notation ‘h.c.’ denotes the Hermitian conjugate.

Discussions of spin in quantum mechanics make use of the Pauli sigma matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Products of these matrices satisfy the identity

$$\sigma^i \sigma^j = \delta^{ij} + i \epsilon^{ijk} \sigma^k.$$

It is convenient to define the linear combinations $\sigma^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$; then

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Fourier Transforms and Distributions

We will often make use of the Heaviside step function $\theta(x)$ and the Dirac delta function $\delta(x)$, defined as follows:

$$\theta(x) = \begin{cases} 0 & x < 0, \\ 1 & x > 0; \end{cases} \quad \delta(x) = \frac{d}{dx} \theta(x).$$

Basic paradigms

$$S = \int d^4x \mathcal{L}(\phi_i, g_i)$$

action

Lagrange density

Fields $\phi_i(x)$ $x=(t, \vec{x})$

define a theory

fundamental parameters
(fine structure constant, electron mass, ...)

- Which fields exist ?

(Fields are the primary objects, not the particles)

- Which symmetries exist ?

(Symmetries contain the structure of $\mathcal{L}$)

In relativistic QFT one always requires that the action is invariant under Lorentz transformations and space-time translation. More precisely,

$$g = (\Lambda, a) \text{ element of the Poincare group} \quad x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu}$$

↑ boost
rotation

translation

or inhomogeneous Lorentz group

The state of a quantum system also transform, i.e.

$$(\Lambda, a) \longmapsto U(\Lambda, a)$$

operator on the Hilbert space

$$|\psi'\rangle = U(\Lambda, a) |\psi\rangle$$

$$\hat{O}' = U(\Lambda, a) \hat{O} U^{-1}(\Lambda, a)$$

Invariance means that $|\langle \psi' | \psi' \rangle| = |\langle \psi | \psi \rangle|$

and $U(\Lambda, a)$ must be a unitary operator (possible up to a phase)

A field operator $O(x)$ is called scalar operator, if

$$U(\Lambda, a) \hat{O}(x) U^{-1}(\Lambda, a) = \hat{O}(\Lambda x + a)$$

$\Rightarrow$ S is invariant, if $\mathcal{L}$ is a scalar operator

The Standard Model of particle physics which describes all observations (except for the existence of dark matter) is based on an amazingly small number of fields and symmetry principles

Fields	Q_i, U_i, D_i	left- and right-handed quarks ($i=1, 2, 3$ for three generations)
	L_i, E_i	left- and right-handed leptons
		left-handed only neutrinos
	ϕ	the Higgs field
	$A_{uc(i)}, A_{sc(i)}, A_{sc(3)}$	Photon $U(1)$
		Gluon $SU(3)$
		W/Z - Boson $SU(2)$
	$g_{\mu\nu}$	the gravitational metric field. There is no evidence yet that this is a quantum field. But there is no reason why it shouldn't be ($\rightarrow$ gravitons)

Symmetries	$SU(3) \times SU(2) \times U(1)$	gauge-symmetry
	strong interaction	
		electromagnetic + weak interactions
	Poincare symmetry	

The rough masses of some particles:

Particle	Mass
neutrinos	$\sim 10^{-2}$ eV
electron	0.5 MeV
Muon	100 MeV
Pions	140 MeV
Proton, Neutron	1 GeV
Tau	2 GeV
W,Z Bosons	80-90 GeV
Higgs Boson	125 GeV

Constructive QFT: « Quantum Physics » Glimm & Jaffe

① Euclidean Axioms

Define QFT in terms of analytical continuation to imaginary time.

Osterwalder - Schrader 1973, 1975

OS0 Analyticity

OS1 Regularity

OS2 Euclidean invariance

OS3 Reflection positivity

OS4 Ergodicity (各态历经)

② Minkowski Space Axioms

Wightman & Haag-Kastler axioms

W1 Covariance: A continuous unitary representation of inhomogeneous Lorentz group $g \rightarrow U(g)$ on the Hilbert space $\mathcal{H}$ of quantum theory states.

W2: Observables: field operators densely defined on $\mathcal{H}$

W3: Locality: f and h are spacelike, $[\phi(f), \phi(h)] = 0$

W4: The vacuum vector is the unique vector

Haag-Kastler axiom focus on the algebraic aspects of the fields, independently of their action on the specific Hilbert space $\mathcal{H}$

Open questions:

Can one find a non-trivial, non-linear QFT in 4-D space-time?

2022 Fields Medal. Hugo Duminil-Copin.

Proof: Triviality of the 4D $\lambda\phi^4$ theory